FWHM Notes

Bandpass responses

The bandpasses we use are often described by central wavelength and Full Width Half Maximum (FWHM). For many filters this is a complete specification but for others it is an approximation.

If $\varphi_j(\lambda)$ is the spectral response for band j then the FWHM is the width of the zone specified by the two (or two of) the points where the value falls to half the maximum. Obviously, the assumption that there is one main peak and a roughly unimodal and even roughly symmetric shape to the response (so that the "central" wavelength is near the peak value) is implicit in this.

A more general way to describe the situation is by the moments of $\varphi_j(\lambda)$:

$$m_0: \quad \int_{-\infty}^{\infty} \varphi_j(\lambda)\, d\lambda$$

$$m_1: \quad \int_{-\infty}^{\infty} \lambda \varphi_j(\lambda)\, d\lambda$$

$$m_2: \quad \int_{-\infty}^{\infty} (\lambda - m_1)^2 \varphi_j(\lambda)\, d\lambda$$

In general, we assume that $\varphi_j(\lambda)$ is normalised so that $m_0=1$. The data on spectral responses are often provided normalised so that the peak value is 1 but the above is better for applying the filter.

The first moment is generally what we mean by the central wavelength so we will denote it as λ_j and call it the "central wavelength". That leaves m_2 as the main criterion for comparing filters which for obvious reasons will be denoted σ_j^2.

Without arguing the toss too much here, I will claim that σ_j^2 is a better statistic to compare bandwidths than FWHM. If two spectral responses have the same values of these three statistics then they tend to behave more similarly than if they just have the same first two and FWHM. FWHM is useful since if detectors are spaced at about the FWHM then there is resolution but little "gap". But often we wish to compare radiometric performance.

Simple Cases

The relationships will only be discussed here for the Uniform distribution, the triangular distribution and the Gaussian distribution.

The uniform case would arise with (say) casi data where a casi band is constructed by aggregating individual 1.8 nm width detector responses. A block of 5 detectors is a uniform filter of width about 9 nm. In this case the FWHM is 9 nm.

Triangular models cover a wide range of spectral responses by available filters and gratings. Hymap filters are supposedly well approximated by triangular functions.

Gaussian responses seem to be good models for Hyperion and maybe others. It is important, therefore, that we investigate the responses of all the instruments such as the ASD (its FWHM is not 1 nm!) and the irradiance devices we use and assess the shape of the filter as well as the FWHM.

Case 1 – Rectangular response

For the rectangular response we have:

$$\varphi_R(\lambda) = \begin{cases} 1/h_R & \left|\lambda - \lambda_j\right| < h_R/2 \\ 1/2h_R & \left|\lambda - \lambda_j\right| = h_R/2 \\ 0 \quad else & \end{cases}$$

It is easy to see that:

$$\int_{-\infty}^{\infty} \varphi_R(\lambda)\, d\lambda = 1$$
$$\int_{-\infty}^{\infty} \lambda \varphi_R(\lambda)\, d\lambda = \lambda_j$$
$$\int_{-\infty}^{\infty} (\lambda - \lambda_j)^2 \varphi_R(\lambda)\, d\lambda = \sigma_R^2 = \frac{h_R^2}{12}$$

Also:

$$FWHM_R = h_R$$
$$\sigma_R = \frac{FWHM_R}{\sqrt{12}}$$

Case 2 – Triangular response

For the triangular response we have:

$$\varphi_T(\lambda) = \begin{cases} \dfrac{4}{h_T^2}\left(h_T/2 - \left|\lambda - \lambda_j\right|\right) & \left|\lambda - \lambda_j\right| < h_T/2 \\ 0 & else \end{cases}$$

It is easy to see that:

$$\int_{-\infty}^{\infty} \varphi_T(\lambda)\, d\lambda = 1$$

$$\int_{-\infty}^{\infty} \lambda\varphi_T(\lambda)\, d\lambda = \lambda_j$$

$$\int_{-\infty}^{\infty} (\lambda-\lambda_j)^2 \varphi_T(\lambda)\, d\lambda = \sigma_R^2 = \frac{h_T^2}{24}$$

Also:

$$FWHM_T = \frac{h_T}{2}$$

$$\sigma_T = \frac{FWHM_T}{\sqrt{6}}$$

Case 2 – Gaussian Response

For the Gaussian case we have:

$$\varphi_G(\lambda) = \frac{1}{\sqrt{2\pi\sigma_G^2}} e^{-\frac{(\lambda-\lambda_j)^2}{2\sigma_G^2}}$$

From which obviously:

$$\int_{-\infty}^{\infty} \varphi_G(\lambda)\, d\lambda = 1$$

$$\int_{-\infty}^{\infty} \lambda\varphi_G(\lambda)\, d\lambda = \lambda_j$$

$$\int_{-\infty}^{\infty} (\lambda-\lambda_j)^2 \varphi_G(\lambda)\, d\lambda = \sigma_G^2$$

The FWHM is defined as the distance between the points where:

$$\varphi_G(\lambda) = \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_G^2}}$$

For the FWHM therefore:

$$FWHM_G = 2\left[2\,Ln2\right]^{1/2} \sigma_G$$

$$\sigma_G = \frac{FWHM_G}{2\left[2\,Ln2\right]^{1/2}}$$

Implications

There are a number of implications from all this. Firstly, if you are going to apply a Gaussian response from an instrument to finer resolved data (eg integrating ASD data to Hymap or Hyperion bands) then simply compute σ_G from the FWHM and integrate between (say) + and – 2.5 σ_G about λ_j. Remember to normalise the filter values to sum to 1.0. It is even easier to apply a Triangular filter.

A second implication is that in the sense that spectral response functions with equal moments up to σ are "equivalent" then the "equivalent" FWHM between the Rectangular, Triangular and Gaussian responses is as follows:

$$
\begin{aligned}
FWHM_T &= \left[\frac{3}{4\,Ln2}\right]^{1/2} FWHM_G \\
&\approx 1.0402\, FWHM_G \\
FWHM_R &= \left[\frac{3}{2\,Ln2}\right]^{1/2} FWHM_G \\
&\approx 1.471\, FWHM_G
\end{aligned}
$$

That is, a casi bandwidth of 9 nm is effectively "equivalent" to a Gaussian FWHM of 6.12 nm or a Hymap FWHM of about 15 nm is effectively "equivalent" to a casi band width of 22 nm.

The FWHM is also effectively "equivalent" and the same magnitude for both Triangular and Gaussian. That is, the rectangular shows most differences with the other two. Knowing the shape of the filter is therefore very important for every instrument.

DLBJ
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